THE DYNAMICS OF COUPLED NONLINEAR OSCILLATORS: FROM RELAXATION OSCILLATORS TO NEURONS

Paul S. Linsay[1,2], *Adam A. Brailove*[1], *Paolo Gaudiano*[3,4], *Andrzej Przybyszewski*[3,5], *Chris Wilson*[1]
[1]Plasma Fusion Center, 175 Albany St., M.I.T., Cambridge, MA 02139, USA
[2]linsay@pfc.mit.edu
[3]Dept. of Cognitive and Neural Systems, 111 Cummington St., Boston University, Boston, MA 02215, USA
[4]gaudiano@cns.bu.edu, [5]przy@cns.bu.edu

ABSTRACT

The dynamics of coupled nonlinear oscillators are extremely rich and important for a large variety of physical, technological, and biological systems. We present results for three different applications: a study of the complex behavior of several coupled electronic relaxation oscillators; a network of electronic neurons designed to simulate the dynamics of a cat retina; and an array of coupled nonlinear oscillators that can perform scene segmentation and pattern recognition.

1. INTRODUCTION

Coupled nonlinear oscillators model a wide variety of major technological and physical systems. Among the natural systems for which coupled-oscillator modelling is important are the Belouzov-Zhabotinsky reaction, electric cardiac conduction, networks of neurons, and turbulent flows. An understanding of phase locking mechanisms is very important for technological devices such as coupled high-power relativistic magnetrons, coupled microwave oscillator drivers for particle accelerators, arrays of semiconductor lasers, arrays of Josephson junctions, and solid-state devices in which sliding charge-density waves occur.

An important and useful property of a nonlinear oscillator is that it can synchronize to another oscillator given the proper coupling. An array of coupled nonlinear oscillators can synchronize to a common frequency even though the frequencies of the free running oscillators are not the same. A good understanding of how networks of coupled nonlinear oscillators phase lock is the underpinning for using these devices in practical applications.

We discuss three different systems of coupled nonlinear oscillators in this paper. In section 2 we describe the complex dynamics of a globally coupled array of relaxation oscillators that have an anti-phase coupling. A ring network of coupled "e-neurons" that model the dynamics of a cat retina is discussed in section 3. Finally in section 4 we present results from a network with local excitation and global inhibition that can do pattern recognition, in this case, scene segmentation of a picture.

2. AN ARRAY OF RELAXATION OSCILLATORS

We first discuss a system of fifteen coupled oscillators and describe some of their properties when coupled "all-to-all". The oscillators were relaxation oscillators modified so that each oscillator could be set to any of 512 distinct initial states in its charge-discharge cycle. A simplified partial schematic of the experiment is shown in figure 1. The oscillators were coupled together in an all-to-all coupling configuration with the strength of the coupling determined by the resistor Rc: there was no coupling when Rc was 0 and maximum coupling when it was large.

The fundamental measurement was the computation of locking time that established when the oscillators were in phase and allowed direct computation of the period and relative oscillator phases. Oscillator zero was designated as the reference oscillator and all measurements were made with respect to it. If the coupling was small, then the oscillators were unlocked and the power spectrum indicated that the system was quasiperiodic. Once the coupling was large enough, the oscillators would always phase-lock to each other to create a periodic state. Just below the critical coupling, the minimum required to achieve phase-locking, there was a region where the system could be quasiperiodic or periodic depending on initial conditions, but there was no coupling that produced chaotic behavior.

The behavior of the transients carries interesting information about the final states and the basin structure. We studied this by measuring the locking time distribution as a function of both the coupling strength and the number of coupled oscillators. The distribution was measured by initializing the oscillators with 1000 random initial conditions and accumulating the locking times. The shape of this distribution varied with the number of oscillators. With small numbers of oscillators, two to four, the distributions tended to be sharply peaked but

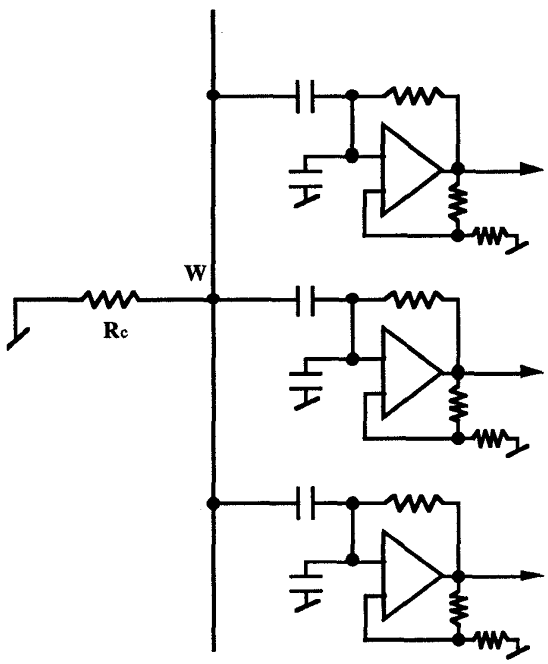

Fig. 1 Simplified partial schematic of a network of relaxation oscillators coupled "all-to-all". Coupling is through the resistor R_c.

gradually developed an exponential background as the number of coupled oscillators increased. Once the number of oscillators was as large as eight, the distribution was purely exponential with no trace of the peaks that appeared with smaller numbers of oscillators. The slope of the exponential was a function of the coupling among the oscillators. Just above the critical coupling the slope varied inversely with the strength of the coupling resistance resulting in very long locking times close to the critical coupling and much shorter times as the coupling strength increased.

The number and character of the final states is also interesting. For N oscillators, the number of states increased from a few near the critical coupling to (N-1)! once the coupling was strong enough. Since this number increases so rapidly, we were only able to verify this directly for seven or fewer oscillators. However, this is the number of permutations of N objects arranged in a circle, and what one expects since no oscillator occupies a preferred position physically in the all-to-all coupling scheme. For three, four, and five oscillators it was also clear that each final state had its own period and locking time distribution. As the numbers of oscillators increase, these distinctions wash out since the different periods become very close to each other and the distributions overlap. We also found that the basins of attraction for the final states were fractal, small changes in initial conditions lead to very different final periodic states. Even though this system exhibits no chaos, it does show a form of sensitive dependence on initial conditions [1, 2]. More details about this experiment can be found elsewhere [3, 4].

3. A SIMULATED CAT RETINA

An electronic circuit consisting of coupled nonlinear oscillators was built to simulate the spatiotemporal processing in a cat retina. Complex behavior recorded *in vivo* from ganglion cells in the cat retina in response to flickering light spots was matched by setting the coupling parameters in the hardware oscillators. An electronic neuron (e-neuron) is composed of four coupled oscillators: three representing the light-driven generator potential of the ganglion cell, the other representing membrane spiking. A 1-D ring of e-neurons reflects the connectivity in the retina: strong neighborhood excitation, and wider inhibition. E-neurons, like retinal ganglion cells, exhibit spontaneous spiking. Driving more than one e-neuron with a sinusoidally modulated input increases regularity in the e-neurons responses, as is found in the retina. We encoded e-neuron activity into single-bit spike trains and found chaotic spontaneous oscillations using close return histograms. The model's behavior gives a new understanding of neurophysiological findings.

As a basic subunit a circuit designed by Keener [5] was used. This analog circuit models Bonhöffer-van der Pol or FitzHugh-Nagumo equations:

$$\frac{dV}{dt} = \frac{1}{\varepsilon}(I_o - I - G(V))$$
$$\frac{dI}{dt} = \beta V - I - V_o$$

where $\varepsilon \in [0.01, 0.5]$ is a positive constant which determines the nonlinearity, G(V) is a piecewise linear function which approximates a cubic polynomial, b and V_0 are constants, V_0 determines resting equilibrium (fixed point or limit cycle), I_0 is an externally modulated input.

By appropriate parameter choice, the circuit can model the ganglion cell generator potential or membrane properties [6] . The membrane properties are characterized by two ion channels, sodium and potassium. The slow potassium current is approximated by a linear function. The sodium current i_{Na} is fast and it has N-shape nonlinear characteristics as a function of the membrane potential.

An e-neuron consists of four resistively coupled circuits. Three of them have small nonlinearity ($\varepsilon = 0.2$) and different eigenfrequencies (1:8:24) and they model a generator potential of the ganglion cell. The fourth circuit models the ganglion cell membrane properties and has a large nonlinearity ($\varepsilon = 0.01$) and 1.6 times higher eigenfrequency than the fastest generator potential.

Figure 2 shows sample traces from a single e-neuron. These model the experimental responses quite well. The complete network exhibits spontaneous oscillations, local excitation of near neighbors, and inhibition of distant neighbors similar to physiological function[7].

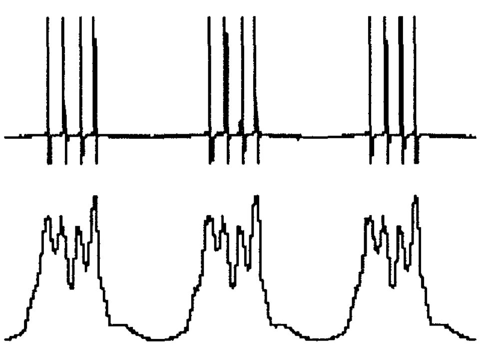

Fig. 2 Membrane potential(upper trace) and generator potential (lower trace) of a single e-neuron

4. SCENE SEGMENTATION BY COUPLED NONLINEAR OSCILLATORS

Humans and other animals analyze and recognize patterns with ease, yet this has proven to be a very hard problem for a computer. One of these problems is scene segmentation, identifying and distinguishing objects in a picture. Current theories suggest that the ability to solve this problem arises from the dynamical properties of spike generating neurons, and specifically from the temporary synchronization of a network of these nonlinear oscillators. In this encoding, each pattern corresponds to a synchronized block of stimulated oscillators, and different patterns correspond to oscillator blocks that are desynchronized from each other.

Developments in nonlinear dynamics make it possible to build a device that can automatically count and locate objects in a high contrast picture in real time [8, 9]. It can be constructed from a network of coupled nonlinear oscillators that are analogs of the FitzHugh-Nagumo equations which crudely model the dynamical properties of a nerve cell. No training of the network is required, all the coupling parameters would be set at the time of its construction. If the fundamental oscillation frequency of the oscillators is chosen correctly, this device would also be able to count and locate moving objects in a scene. If the inputs to the oscillators are made adaptive, the device can be made sensitive to changes in a static scene that is under surveillance. Such a device can be easily built from analog circuit components and eventually as an integrated circuit.

To build such an array, the oscillators are locally coupled to their nearest neighbors while a single global inhibitor receives input and sends output to all the oscillators. The local coupling permits oscillators belonging to the same object to synchronize while the global inhibitor forces groups of oscillators that belong to different objects to desynchronize. Both types of coupling are necessary for scene segmentation. If the global inhibition were not present, oscillators corresponding to an object would synchronize but there would be random phase differences among the different objects, which would frequently lead to accidental synchronization. If there were only a global coupling, the geometry would be lost and there would be no way of distinguishing among the objects.

There is a complete theory for the locally-excited-globally-inhibited-oscillator-network (LEGION) [8, 9] that specifies the nonlinear oscillator properties, the local and global coupling strengths, and predicts the behavior of a network. Most importantly, the theory shows that scene segmentation is an intrinsic and robust property of the model, exists for a large class of initial data, and is not sensitive to moderate changes of the parameters.

A LEGION array is automatically parallel and naturally designed as an analog circuit making it very attractive for realtime problems. Although it was first implemented on a digital computer, applying the algorithm to an n x n pixel array involves simultaneously integrating $2n^2$ coupled nonlinear differential equations. The computing power required to do this becomes very large, very fast as the resolution of a scene increases. A more interesting alternative would be to implement the analog circuit on a VLSI chip.

The design discussed here is the simplest possible. It has fixed connection weights between neighbors and is suitable for a picture that has only black and white pixels. A modest modification of the input coupling can eliminate sensitivity to small disconnected regions of 1 or 2 pixels that could clutter a picture. Scene segmentation can be applied to a grey-scale picture if the near neighbor couplings are made inversely proportional to pixel value differences. Both these possibilities can be implemented in hardware in a straightforward way. Neither the simple circuit nor either of these variations requires the circuit to do any "learning."

LEGION networks have a wide variety of possible uses, anything where machine vision is desired. They should be useful for automatic targeting, surveillance, or similar military purposes. They could serve as an automatic analyzer of a visual environment, such as data from remote sensing, and a front-end to a pattern recognizer. In computer simulation they have been used to segment MR and CT scan images. With some straightforward extensions, LEGION networks may be used to segregate patterns embedded in time, such as acoustic signals, and identify the source of each temporal signal.

We have built a small 2x4 array to perform scene segmentation using circuits similar to those used to simulate the cat retina. The coupling is fixed strength near neighbor coupling with a global inhibitor to desynchronize disconnected "objects". Near neighbors synchronize if they are simultaneously excited by an external stimulus. Disconnected neighbors that are excited, oscillate out of phase. It is capable of "identifying" up to 4 distinct objects. There appears to be no obstacle to building large arrays that will do scene segmentation in real time.

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